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Pattern analysis in a benthic bacteria-nutrient system

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We study steady states in a reaction-diffusion system for a benthic bacterianutrient model in a marine sediment over 1D and 2D domains by using Landau reductions and numerical path following methods. We point out how the system reacts to changes of the strength of food supply and ingestion. We find that the system has a stable homogeneous steady state for relatively large rates of food supply and ingestion, while this state becomes unstable if one of these rates decreases and Turing patterns such as hexagons and stripes start to exist. One of the main results of the present work is a global bifurcation diagram for solutions over a bounded 2D domain. This bifurcation diagram includes branches of stripes, hexagons, and mixed modes. Furthermore, we find a number of snaking branches of stationary states, which are spatial connections between homogeneous states and hexagons, homogeneous states and stripes as well as stripes and hexagons in parameter ranges, where both corresponding states are stable. The system under consideration originally contains some spatially varying coefficients and with these exhibits layerings of patterns. The existence of spatial connections between different steady states in bistable ranges shows that spatially varying patterns are not necessarily due to spatially varying coefficients. The present work gives another example, where these effects arise and shows how the analytical and numerical observations can be used to detect signs that a marine bacteria population is in danger to die out or on its way to recovery, respectively. We find a type of hexagon patch on a homogeneous background, which seems to be new discovery. We show the first numerically calculated solution-branch, which connects two different types of hexagons in parameter space. We check numerically for bounded domains whether the stability changes for hexagons and stripes, which are extended homogeneously into the third dimension. We find that stripes and one type of hexagons have the same stable range over bounded 2D and 3D domains. This does not hold for the other type of hexagons. Their stable range is shorter for the bounded 3D domain, which we used here. We find a snaking branch, which bifurcates when the hexagonal prisms loose their stability. Solutions on this branch connects spatially between hexagonal prisms and a genuine 3D pattern (balls).