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Synchronization patterns in Stuart-Landau networks: a reduced system approach

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We study networks with coupled phase and amplitude dynamics. In particular, we investigate a ring network of Stuart-Landau oscillators with two types of coupling: S^1 symmetry-breaking and S^1 symmetry-conserving. Symmetry-breaking coupling enables the appearance of nontrivial spatio-temporal patterns, such as amplitude chimeras, which exhibit domains of coherent (synchronized) and incoherent (desynchronized) dynamics with respect to the amplitude, but the phases are always correlated. These states are long-living transients. We show that amplitude chimeras can arise even in small networks of only 12 elements. For symmetry-preserving coupling we find cluster synchronization. We show that the dimension of the dynamical system can be substantially reduced, which facilitates the simulations, by projecting the system onto the subspace corresponding to the unstable eigenvalues of the linear part of the network dynamics. First, we decompose the dynamic variables into a mean-field part corresponding to motion on the synchronization manifold $\mathbf{z_s}$ and a synchronization error $\mathbf{e}$. The linear part of the network dynamics, after transforming to Jordan normal form, can be split into a low-dimensional synchronization manifold which is associated with the unstable directions of the fixed point at the origin from which the synchronized limit cycle oscillations have bifurcated, (and hence with those eigenvalues which have positive real parts) and a transverse subspace of the synchronization error, (which is associated with negative real parts) leading to an asymptotically damped synchronization error. The reduced system consists of the subspace of unstable eigenvalues. We present simulations for a network of $N = 12$ nodes which can be reduced to a 3-dimensional system with this method. We show that the asymptotic collective behavior of the network is determined by the subspace of unstable eigenvalues, while the dynamics in the remaining subspace is asymptotically damped out. In this way we reduce the network of Stuart-Landau oscillators to a lower-dimensional system. To validate this method, we apply a back transformation to the reduced system, and compare the solutions with the full dynamics.