* Electronic Address: szmolyan@tuwien.ac.at
[1] Technische Universität Wien

Blow-up analysis of regularizations of nonsmooth systems

Peter Szmolyan[1*]

Nonsmooth vector fields X_0 arise frequently in applications. Many results and methods from the theory of smooth dynamical systems do not directly carry over to these more singular situations. One possible remedy is to consider nonsmooth problems as $\varepsilon \to 0$ limits of smooth vector fields X_ε. The regularized problems X_ε are typically obtained by either including regularizing effects in the underlying model or by some suitable mathematical regularization procedure. A prototypical class of such problems are piecewise smooth vector fields which are smooth except on a co-dimension one hypersurface S_0. The main issues are how to properly define the dynamics on S_0 and to classify dynamical effects associated with the discontinuity. A well known phenomenon in these systems is "sliding", that is motion along S_0 if the vector field X_0 points towards S_0 on both sides of that hypersurface. More recently, interesting links between regularizations of piecewise smooth vector fields and fast–slow systems have emerged. It has been established by several authors that in certain situations sliding is closely related to the existence of a slow manifold S_ε of X_ε close to S_0. It became apparent that some well known phenomena in fast–slow systems, e.g. folds and canards associated with loss of normal hyperbolicity, have certain close analogs in piecewise smooth problems. In the talk I will survey some of these results and show that the blow-up method – which has proven to be a powerful tool in the context of fast-slow systems – can also be used effectively in the analysis of regularizations of nonsmooth problems.