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Interface layer of a two-component Bose-Einstein condensate

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We study the behaviour of the wave functions of a two-component Bose-Einstein condensate near the interface, in the case of segregation, in the Thomas-Fermi limit. In the direction orthogonal to the interface, possibly after a rescaling, this should be governed by an energy minimizing solution of the problem:

$$\begin{cases} -av_1'' + v_1^3 - v_1 + \Lambda v_2^2 v_1 = 0, \\ \\ -v_2'' + v_2^3 - v_2 + \Lambda v_1^2 v_2 = 0, \end{cases} \tag{1}$$

$$(v_1, v_2) \to (0, 1) \text{ as } z \to -\infty, \quad (v_1, v_2) \to (1, 0) \text{ as } z \to +\infty, \tag{2}$$

where $\Lambda > 1$ represents the intercomponent repulsive strength and $a > 0$ is a fixed parameter. Using singular perturbation arguments, we construct a solution to the above problem with monotone components, while providing a very detailed componentwise description of the limiting behaviour $v_1 v_2 \to 0$ in the case of the strong segragation regime, that is $\Lambda \to \infty$. Our analysis is based on the nondegeneracy of the inner (or blow-up) profile which is a special entire solution of

$$aV_1'' = V_1 V_2^2, \quad V_2'' = V_2 V_1^2, \tag{3}$$

as well as on the nondeneracy of the outer profiles which satisfy the Allen-Cahn type of problems that give the leading order behaviour of solutions to (1)-(2) as $\Lambda \to \infty$. Furtheremore, we prove that the constructed solution is nondegenerate in the natural sense: the linearized operator of (1)-(2) about the solution, in $L^2(\mathbb{R}) \times L^2(\mathbb{R})$, has zero as a simple eigenvalue at the bottom of its spectrum and the rest of the spectrum is contained in an interval of the form $[c, \infty)$ with $c > 0$ independent of $\Lambda \gg 1$. Moreover, we prove that there is a unique solution (modulo translations) to (1)-(2) with positive components such that at least one is monotone. In turn, this implies that the constructed solution is a minimizer of the associated energy to (1)-(2). We can then exploit our detailed estimates from the singular perturbation analysis and provide an asymptotic expression for its energy, which is in agreement with that predicted in the physics literature. Finally, our existence and uniqueness results for (1)-(2) can be combined to give a new proof of the uniqueness (modulo translations and scalings) of the blow-up profile in (3). This is a joint work with A. Aftalion.