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An implicit function theorem for singularly perturbed problems

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Many analytical treatments of singularly perturbed problems fit into the following scheme: First one constructs, in a rather formal and ad hoc manner, a family (parametrized by the singular perturbation parameter) of so-called approximate solutions, i.e. of functions which solve the problem approximately. Usually those functions have a certain structure (internal and boundary layers, spikes etc.) which is interesting for applications, and one believes that for all sufficiently small singular perturbation parameters there exists an exact solution to the problem close to the approximate solution. And second, one proves that really for all sufficiently small perturbation parameters there exists an exact solution to the problem close to the approximate solution. In this talk we will present sufficient conditions in terms of the problem data and of the approximate solutions which guarantee existence and local uniqueness of an exact solution close to an approximate solution. Moreover, we derive an a priori estimate for the distance between the exact and the approximate solutions. We will work in a completely abstract setting, but we will mention applications of our abstract results to singularly perturbed semilinear elliptic and parabolic problems with (probably non-monotone) boundary layer solutions and spike solutions as well as to problems with multidimensional singular perturbation parameter. This is joint work with V. F. Butuzov, N. N. Nefedov and O. E. Omel'chenko.