* Electronic Address: `lermanl@mm.unn.ru`
[1] Lobachevsky State University of Nizhny Novgorod

Geometry and dynamics of slow fast Hamiltonian systems

Lev Lerman[1*]

We introduce some geometric tools needed to describe slow-fast Hamiltonian systems on smooth manifolds. We start with a smooth bundle $p : M \to B$ where (M, ω) is a C^∞-smooth presymplectic manifold with a closed constant rank 2-form ω and (B, λ) is a smooth symplectic manifold. The 2-form ω is supposed to be compatible with the structure of the bundle, that is, the bundle fibers are symplectic manifolds with respect to the 2-form ω, the distribution on M generated by kernels of ω is transverse to the tangent spaces of the leaves and the dimensions of the kernels and of the leaves are supplementary. This allows one to define a symplectic structure $\Omega_\varepsilon = \omega + \varepsilon^{-1} p^*\lambda$ on M for any positive small ε, where $p^*\lambda$ is the lift of the 2-form λ to M. Given a smooth Hamiltonian H on M one gets a slow-fast Hamiltonian system with respect to Ω_ε. We define a slow manifold SM for this system. Assuming SM is a smooth submanifold, we define a slow Hamiltonian flow on SM. After that we consider singularities of the restriction of p to SM. We show that if $\dim M = 4$, $\dim B = 2$ and the Hamilton function H is generic, then the behavior of the system near a singularity of fold type is described, to the main order, by the equation Painlevé-I, and if this singularity is a cusp, then the related equation is Painlevé-II. The dynamical patterns will be illustrated for the case of the Duffing type equation with slow periodically varying parameters.