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Mathematical modelling and simulation of the evolution of plaques in blood vessels

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This lecture deals with the mathematical modelling of formation and evolution of plaques in arteries, one of the main causes for the blockage of blood flow. Plaque rupture and spread of torn-off material may cause closures in the down-stream vessel system and lead to ischaemic brain or myocardial infarctions. The arising system of biochemical and biophysical processes is huge and complex. We focus on an important sub-system: the transport of immune cells in large blood vessels, their transition through the endothelial layer and their interactions with the vessel walls, causing changes of the volume and mechanical properties and leading to plaque formation. The following topics will be covered in this talk:

- Derivation of the model system as a mechano-chemical fluid-structure interaction problem
- Numerical algorithms developed and used to simulate the dynamics of this interaction: The arbitrary Lagrangian Eulerian method (ALE) is chosen to solve the systems numerically.
- Results of simulations of the plaque formation for realistic systems parameters: The evolving structures are matching clinical observations: The time scale of the formation is in the simulation of comparable order as in reality. Simulations of plaques demonstrate pattern formation, which was not expected, but can be observed.

Changes of the blood perfusion is also leading to changes in the supply in particular with oxygen, which is an important factor in the dynamics of inflammation. Hypoxia is responsible for changes in water permeability of cell walls, leading to osmotic effects and as a consequence to swelling of cells and of tissue. A survey on results for swelling of a single cell is presented. The reported results are based on joint research of the authors of the following references.

References

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