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Spiral wave chaos: Tiling, local symmetries, and asymptotic freedom

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Excitable systems can generate dynamics ranging from solitary waves in 1D to spiral/scroll wave chaos in 2D/3D. Complex spatiotemporally chaotic dynamics featuring spiral waves are associated with phenomena such as cardiac arrhythmias (e.g., fibrillation) and seizures (epilepsy). Understanding the nature of spatiotemporal chaos in excitable systems therefore is not only of fundamental interest, but also of high practical importance. This talk will give an overview of recent progress in understanding the dynamical mechanisms that initiate and maintain spiral wave chaos featuring multiple interacting spiral waves that repeatedly break up and merge. Periodic orbit theory, which aims to describe chaotic dynamics using the properties of unstable periodic solutions embedded in the chaotic attractor, produced a lot of insight into the dynamics of low-dimensional systems, starting with the work of Poincare on celestial mechanics. Recently, a similar approach has been applied rather successfully to spatiotemporal chaos in a range of systems (complex Ginzburg-Landau, Kuramoto-Sivashinsky, and Navier-Stokes equation). In excitable systems, however, it fails rather spectacularly due to a special property of spiral waves: they have extremely short spatial correlations. Although it is tempting to associate the relevant length scale with the wavelength of a spiral wave, the former is instead defined by the width of the adjoint eigenfunctions associated with the dominant modes of the linearization. For typical models of excitable dynamics these eigenfunctions are exponentially localized around the spiral core, with the width much smaller than the wavelength. Hence, interaction between two spiral waves falls off exponentially, and the dynamics of individual spirals become effectively independent once the separation between the spiral cores exceeds this length scale (spiral waves become asymptotically free). As a result, typical multi-spiral states break the global Euclidean symmetry of the problem, but respect local symmetries (translations and rotations in 2D). Local symmetries imply that time-periodic solutions are extremely rare due to the slow relative drift in the position and orientation of individual spirals. This drift can be understood by partitioning the domain into tiles, each of which supports a single spiral wave. The dynamics of each spiral can then be understood completely based on the shape of the corresponding tile and the position of the spiral core. This formalism produces a number of specific predictions that are fully supported by numerical simulations and offers a novel way to understand and describe spiral wave chaos.