Rigidity and Deformation

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Abstract

Download a triangle-mesh model of a 3D bunny, cut a stick for every edge, and attach them together with a flexible joint at each vertex to re-create the model's one-skeleton. Would it stand up or collapse? Bet on "stand up" — Herman Gluck proved in 1975 that almost all such triangulated one-skeletons are rigid. There are a few examples of non-rigid polyhedra; that is, there is a motion under which the edge lengths remain fixed but the dihedral angles change.

What if, instead of fixing the edge lengths, we fixed the dihedrals? Are there motions which fix the dihedrals but allow the lengths to change? We show an analog of Gluck's theorem, that almost all polyhedra are "dihedral-rigid".

Who cares? Well, deformation is the opposite of rigidity. What can rigidity—and the examples of non-rigidity—tell us about how we can parameterize, measure and control the deformations of a mesh? Parameterizing deformations by edge length turns out to be a bad idea, but we demonstrate that there is reason to be much more hopeful about parameterizing meshes by their dihedrals.

This is work with Carlos Rojas.

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This is an extended abstract of a presentation given at EuroCG'18. It has been made public for the benefit of the community and should be considered a preprint rather than a formally reviewed paper. Thus, this work is expected to appear eventually in more final form at a conference with formal proceedings and/or in a journal.