Stumbling around in the dark: lessons from everyday mathematics

Ursula Martin
University of Oxford

Happy Anniversary, CADE

10 things to be proud of!

Large machine math proofs are feasible

Thomas Hales, associate professor of mathematics, demonstrates his solution to the Kepler conjecture, a problem that mathematicians have been wrestling with since 1611. Tennis balls courtesy of the Varsity Tennis Club. Photo by Bob Kalmbach

… professional engineering projects

Flyspeck, proved Kepler's conjecture, started 2003, completed 2014

Credits

- Project Director: Thomas Hales
- Project Managers: Ta Thi Hoai An, Mark Adams
- HOL Light libraries and support: John Harrison,
- Isabelle Tame Graph Classification: Gertrud Bauer, Tobias Nipkow,
- Chief Programmer: Alexey Solovyev,
  - Nonlinear inequalities: Victor Magron, Sean McLaughlin, Roland Zumkeller,
  - Linear Programming: Steven Obua,
  - Microsoft Azure Cloud support: Daron Green, Joe Pleso, Dan Synek, Wenming Ye,
- Chief Formalizer: Hoang Le Truong,
  - Text formalization: Jason Rute, Dang Tat Dat, Nguyen Tat Thang, Nguyen Quang Truong, Tran Nam Trung, Trieu T Vuong Anh Quyen,
- Student Projects: Catalin Anghel, Matthew Wampler-Doty, Nicholas Volker, Nguyen Duc Tam, Nguyen Duc Thinh, Vu Q
- Proof Automation: Cezary Kaliszyk, Josef Urban,
- Editing: Erin Susick, Laurel Martin, Mary Johnston,
- External Advisors and Design: Freek Wiedijk, Georges Gonthier, Jeremy Avigad, Christian Marchal,
- Institutional Support: NSF, Microsoft Azure Research, William Benter Foundation, University of Pittsburgh, Radboud Ur Math (VAST), VIASM.

… replicable

20 September 2012 - *Mathematical Components*

Feit thomson proved in coq

Feit-Thompson theorem has been totally checked in Coq

Thursday 20 September 2012, 18:16. We received following mail from Georges Gonthier (see below).It concludes the proof in Coq of the Feit-Thompson theorem. This theorem, also named the Odd Order Theorem, is the first main result in the classification of finite groups.This work was achieved by the team formed by addressees of Georges' mail, team strongly
effort (a
theorem
proved
Laurent

From Laurent Théry
Date: Thursday 20 September 2012, 20:24
Re: [Coqfinitgroup-commits] r4105 – trunk
Hi,

Just for fun

Feit Thompson statement in Coq:

Theorem Feit_Thompson (gT : finGroupType) (G : {group gT}) : odd #|G| -> solvable G.

How is it proved?

You can see only green lights there:

http://ssr2.msr-inria.inria.fr/~jenkins/current/progress.html

and the final theory graph at:

http://ssr2.msr-inria.inria.fr/~jenkins/current/index.html

How big it is:

Number of lines ~ 170 000
Number of definitions ~15 000
Number of theorems ~ 4 200
Fun ~ enormous!

— Laurent

Rechercher ok Le CNRS | Annuaires | Mo

Institut des sciences de l'information et de leurs interactions

précédente

… recognised

Coq récompensé par l'Association for Computing Machinery

L'équipe de développement du système Coq se voit décerner le prix "Software System Award" de l'ACM (Association for Computing Machinery). Ce prix récompense un travail collectif mené sur une très longue durée, auquel ont participé de nombreux chercheurs de laboratoires INS2I et d'équipes Inria. La cérémonie de remise du prix aura lieu le 21 juin 2014 à San Francisco.

Le prix "Software System Award", plus haute distinction de l'ACM en matière de logiciel, récompense une institution ou des individus pour le développement d'un système logiciel particulièrement influent par sa contribution conceptuelle et/ou son acceptabilité commerciale. Parmi les précédents récipiendaires figurent par exemple Unix, TeX, TCP/IP, Java, etc.

Coq est un logiciel permettant à la fois la production de programmes informatiques certifiés et la vérification de théorèmes mathématiques. Il est aujourd'hui développé par l'équipe PI.R2 d'Inria au sein du laboratoire PPS (CNRS/Université Paris Diderot-Paris 7).

Ce succès collectif honore l'ensemble de l'équipe des développeurs de Coq et notamment les neuf récipiendaires du Software System Award, parmi lesquels cinq chercheurs travaillant dans des laboratoires de l'INS2I :

- Bruno Barras (LIX), Inria Saclay/École Polytechnique ;
- Yves Bertot, Inria Sophia Antipolis-Méditerranée ;
- Pierre Castéran (LaBRI), Université de Bordeaux ;
- Thierry Coquand, Université de Gothenburg ;
- Jean-Christophe Filliâtre (LRI), CNRS/Inria Saclay - Ile-de-France ;
- Hugo Herbelin (PPS), Inria Paris - Rocquencourt ;
- Gérard Huet, Inria Paris - Rocquencourt ;

Moshe Vardi shared a link.

3 May

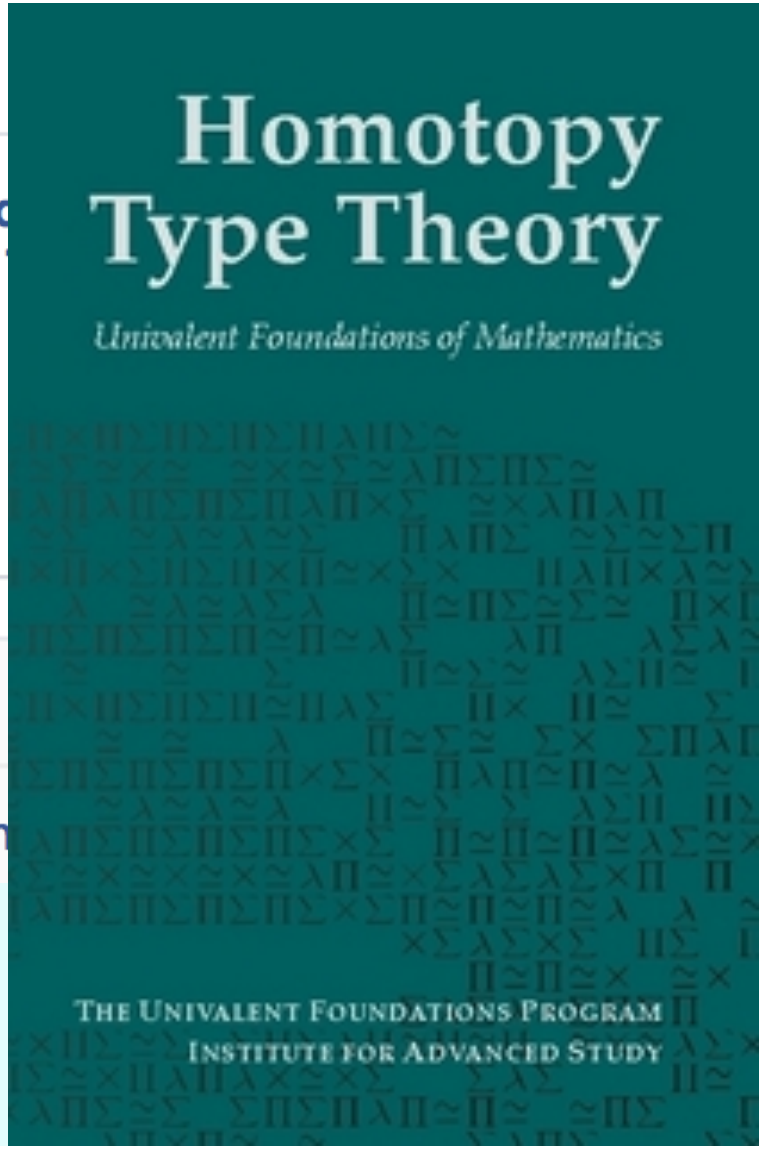
… rewarded

Press Release: Carnegie Mellon Awarded $7.5 Million Department of Defense Grant To Reshape Mathematics

Press Release: Carnegie Mell
Department of Defense Grant
www.cmu.edu

Like · Comment · Share

Hanne Gottliebsen, Valeria De Paiva, Lawrence Paulson



.... have industrial impact

PROGRAMMING PRINCIPLES, LOGIC AND VERIFICATION GROUP

DEPARTMENT OF COMPUTER SCIENCE

UCL

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Richard Bornat (Visiting Professor)

James Brotherston (Senior Lecturer / EPSRC Research Fellow)

Byron Cook (Professor of Computer Science, joint appointment with Amazon)

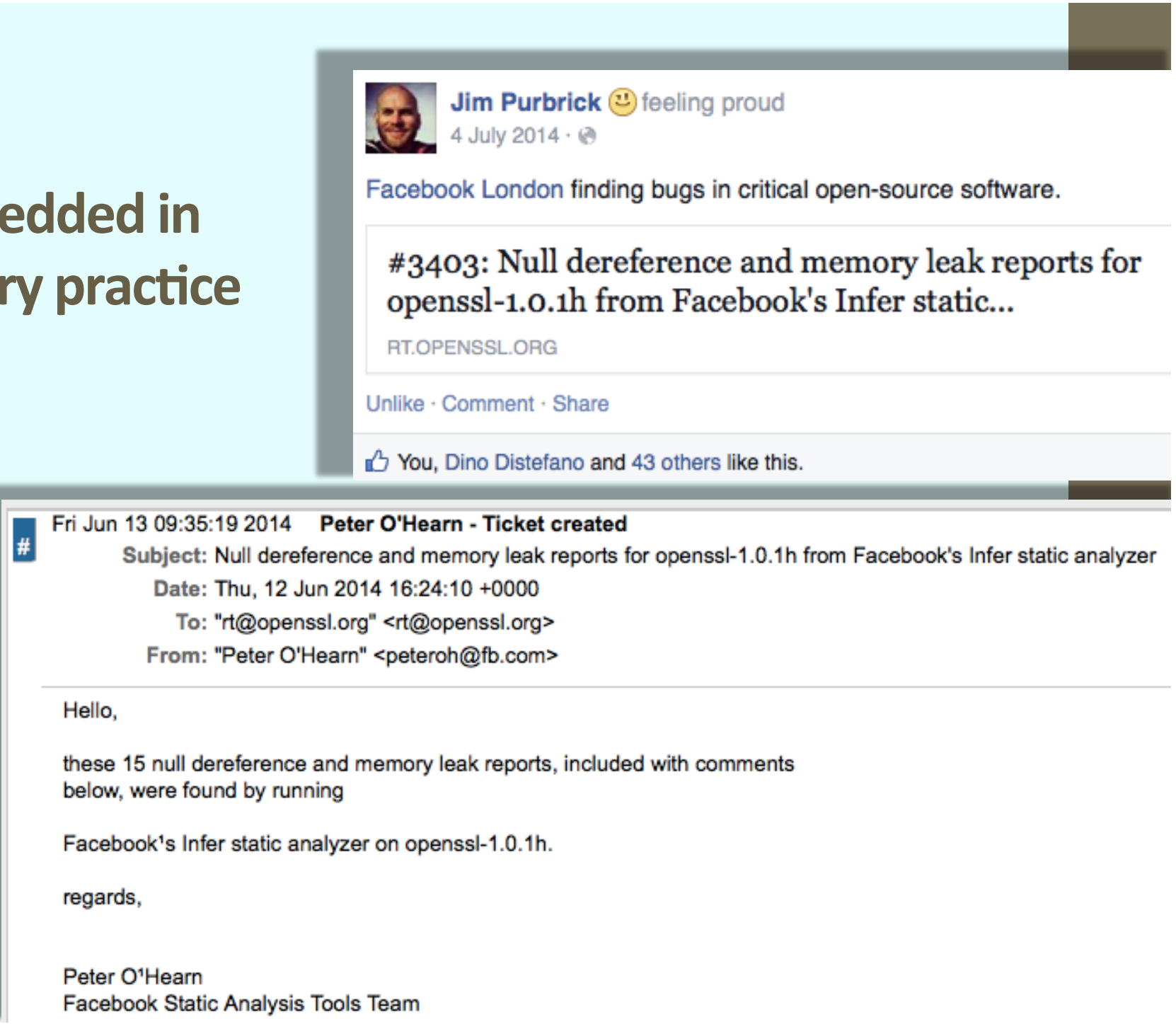
Peter O' Hearn (Professor of Computer Science, on leave at Facebook, part-tim

Robin Hirsch (Professor of Mathematical Foundations of Computing)

Juan Antonio Navarro Pérez (Lecturer, on leave at Google)

David Pym (Professor of Information, Logic, and Security, Head of Group)

.. embedded in industry practice



Jim Purbrick feeling proud
4 July 2014 ·

Facebook London finding bugs in critical open-source software.

#3403: Null dereference and memory leak reports for openssl-1.0.1h from Facebook's Infer static...

RT.OPENSSL.ORG

Unlike · Comment · Share

You, Dino Distefano and 43 others like this.

Fri Jun 13 09:35:19 2014 **Peter O'Hearn - Ticket created**

Subject: Null dereference and memory leak reports for openssl-1.0.1h from Facebook's Infer static analyzer
Date: Thu, 12 Jun 2014 16:24:10 +0000
To: "rt@openssl.org" <rt@openssl.org>
From: "Peter O'Hearn" <peteroh@fb.com>

Hello,

these 15 null dereference and memory leak reports, included with comments
below, were found by running

Facebook's Infer static analyzer on openssl-1.0.1h.

regards,

Peter O'Hearn
Facebook Static Analysis Tools Team

…acceptable to mathematicians…

Gowers's Weblog

Mathematics related discussions

Recent news concerning the Erdos discrepancy problem

The problem is to show that if (ϵ_n) is an infinite sequence of ± 1s, then for every C there exist d and m such that $\sum_{i=1}^{m} \epsilon_{id}$ has modulus at least C. This result is straightforward to prove by an exhaustive search when $C = 2$. One thing that the Polymath project did was to discover several sequences of length 1124 such that no sum has modulus greater than 2, and despite some effort nobody managed to find a longer one. That was enough to convince me that 1124 was the correct bound.

However, the new result shows the danger of this kind of empirical evidence. The authors used state of the art SAT solvers to find a sequence of length 1160 with no sum having modulus greater than 2, and also showed that this bound is best possible. Of this second statement, they write the following: "The negative witness, that is, the DRUP unsatisfiability certificate, is probably one of longest proofs of a non-trivial mathematical result ever produced. Its gigantic size is comparable, for example, with the size of the whole Wikipedia, so one may have doubts about to which degree this can be accepted as a proof of a mathematical statement."

I personally am relaxed about huge computer proofs like this. It is conceivable that the authors made a mistake somewhere, but that is true of conventional proofs as well. The paper is by Boris Konev and Alexei Lisitsa and appears here.

… acceptable to mathematicians

Gowers's Weblog

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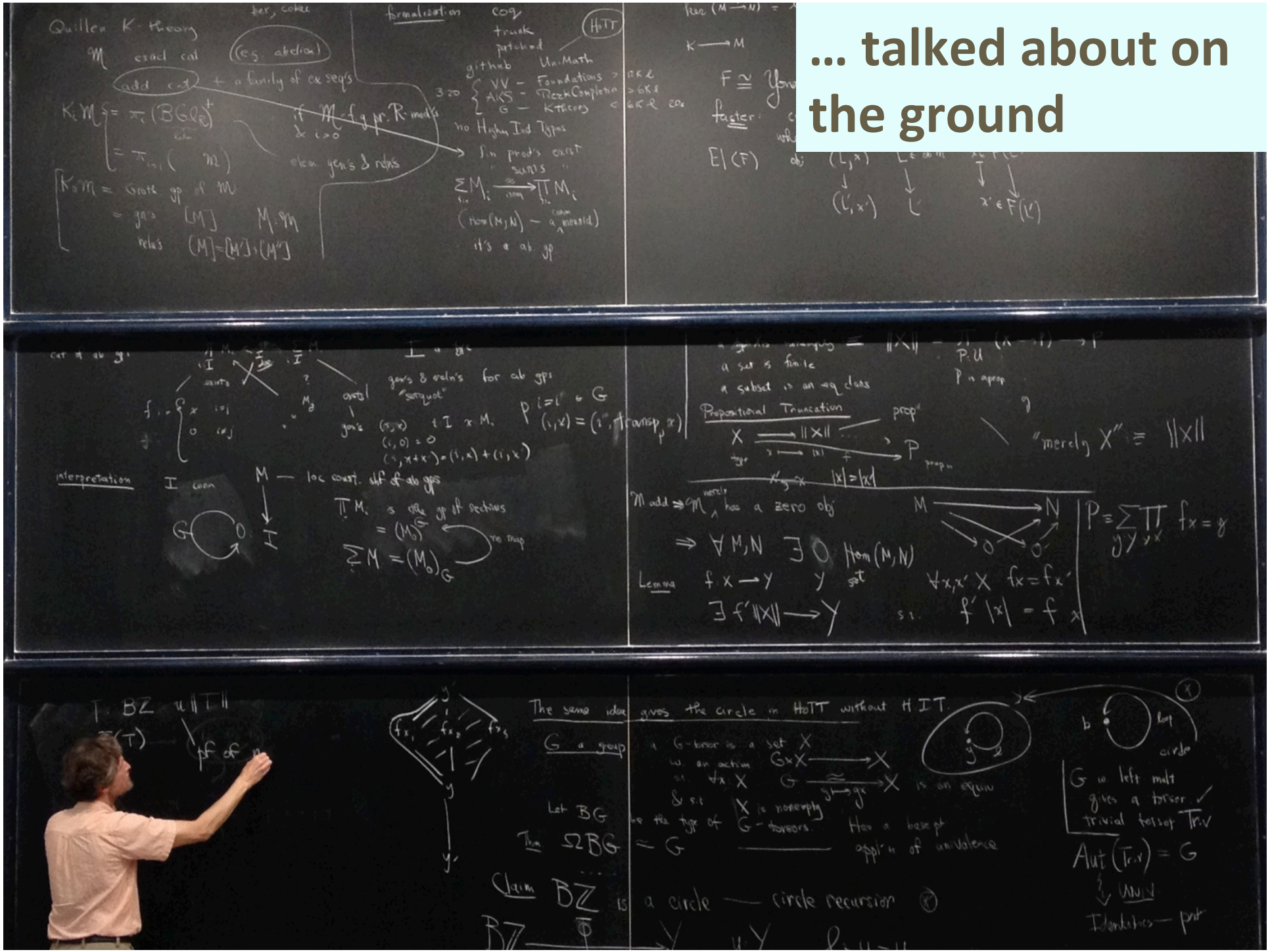


… talked about on the ground

…and from the mountain tops

Lurie: I would like to see a computer proof verification system with an improved user interface, something that doesn't require 100 times as much time as to write down the proof. Can we expect, say in 25 years, widespread adoption of computer verified proofs?

Tao: I hope [we will eventually be able to verify every new paper by machine.]. Perhaps at some point we will write our papers… directly in some formal mathematics system.

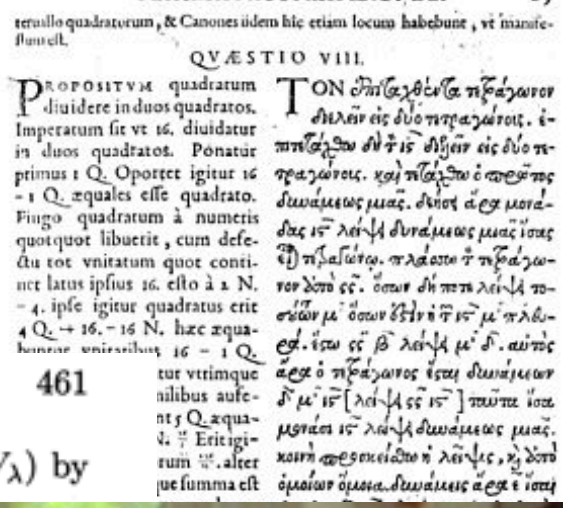
Simon Donaldson, Maxim Kontsevich, Jacob Lurie, Terence Tao, Richard Taylor: award of $3 million Breakthrough Prizes, 2014

Mathematical practice: the mathematicians speak

Mathematical products

Fermats theorem conjectured 1637

Proved Andrew Wiles 1995



Arithmeticorum Lib. II. 85

terullo quadratorum, & Canones iidem hic etiam locum habebunt, vt manifeſtum eſt.

QVÆSTIO VIII.

PROPOSITVM quadratum diuidere in duos quadratos. Imperatum ſit vt 16. diuidatur in duos quadratos. Ponatur primus 1 Q. Oportet igitur 16 - 1 Q. æquales eſſe quadrato. Fingo quadratum à numeris quotquot libuerit, cum defectu tot vnitatum quot continet latus ipſius 16. eſto à 2 N. - 4. ipſe igitur quadratus erit 4 Q. + 16. - 16 N. hæc æqua-

ΤΟΝ δοθέντα τετράγωνον διελεῖν εἰς δύο τετραγώνους. ὑποτεθάχθω δὲ τὸ ις̄ δεῖξαι εἰς δύο τετραγώνους. καὶ τεθάχθω ὁ τοῦ πρώτου δυνάμεως μονάς. δεῖσα ἄρα μονάδας ις̄ λείψει δυνάμεως μοναῖς ἴσας ἐστὶ πλευρῷ. πλάσσω τὸ πλευρὸν δοτὸ ς̄ς̄. ὅσων δὴ πέπε λείψει τοσούτων μ̄. ὅσων δέδηγ ὁ τοῦ ις̄ μ̄. πλευρὰ ἐστὶ μ̄. ἔστω ς̄ς̄ β̄ λείψει μ̄ δ̄. αὐτὸς ἄρα ὁ πλευρὸς ἔσται δυνάμεων δ̄ μ̄ ις̄ [λείψει ς̄ς̄ ις̄] πλῶτα ἴσα μοναῖς ις̄ λείψει δυνάμεως μιᾶς. καὶ πρὸς εἰδῶ τὸ λεῖψις, καὶ δοτὸ ὁμοίων ὁμοια. δυνάμεις ἄρα ε ἴσαις

In the Selmer case we make an analogous definition for $H^1_{\mathrm{Se}}(\mathbf{Q}_p, W_\lambda)$ by replacing V_λ by W_λ, and similarly in the str… the fact that there is a natural isomorphism

$$H^1(\mathbf{Q}_p, V_\lambda) \to \mathrm{Ext}^1_{k[}$$

where the extensions are computed in the cate… Galois action. Then $H^1_f(\mathbf{Q}_p, V_\lambda)$ is defined … which is the inverse image of $\mathrm{Ext}^1_{\mathrm{fl}}(G, G)$, the… gory of finite flat commutative group schemes… (unique) finite flat group scheme over $\mathbf{Z}_p$ asso… extensions in the inverse image even correspon… more details and calculations see [Ram].

For q different from p and $q \in \mathcal{M}$ we h… case (A) there is a filtration by D_q entirely … write this $0 \subset W_\lambda^{0,q} \subset W_\lambda^{1,q} \subset W_\lambda$ and we set…

$$H^1_{D_q}(\mathbf{Q}_q, V_\lambda) = \begin{cases} \ker : H^1(\mathbf{Q}_q, V_\lambda) \\ \quad \to H^1(\mathbf{Q}_q, W_\lambda / W_\lambda^{0,q}) \oplus \\ \ker : H^1(\mathbf{Q}_q, V_\lambda) \\ \quad \to H^1(\mathbf{Q}_q^{\mathrm{unr}}, V_\lambda) \end{cases}$$

Creativity

I used to start trying to find patterns. I tried doing calculations which explain some little piece of mathematics. I tried to fit it in with some previous broad conceptual understanding of some part of mathematics that would clarify things. Sometimes that would involve [looking at references] to see how it's done. Sometimes it was a question of modifying things a bit, doing a little extra calculation. And sometimes I realized that nothing that had ever been done before was any use at all. Then I just had to find something completely new; it's a mystery where that comes from..

Andrew Wiles, 2000, on proving Fermat

Creativity and slog

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Andrew Wiles, 2000, on proving Fermat

… a journey through a dark unexplored mansion. You enter the first room of the mansion and it's completely dark. …..

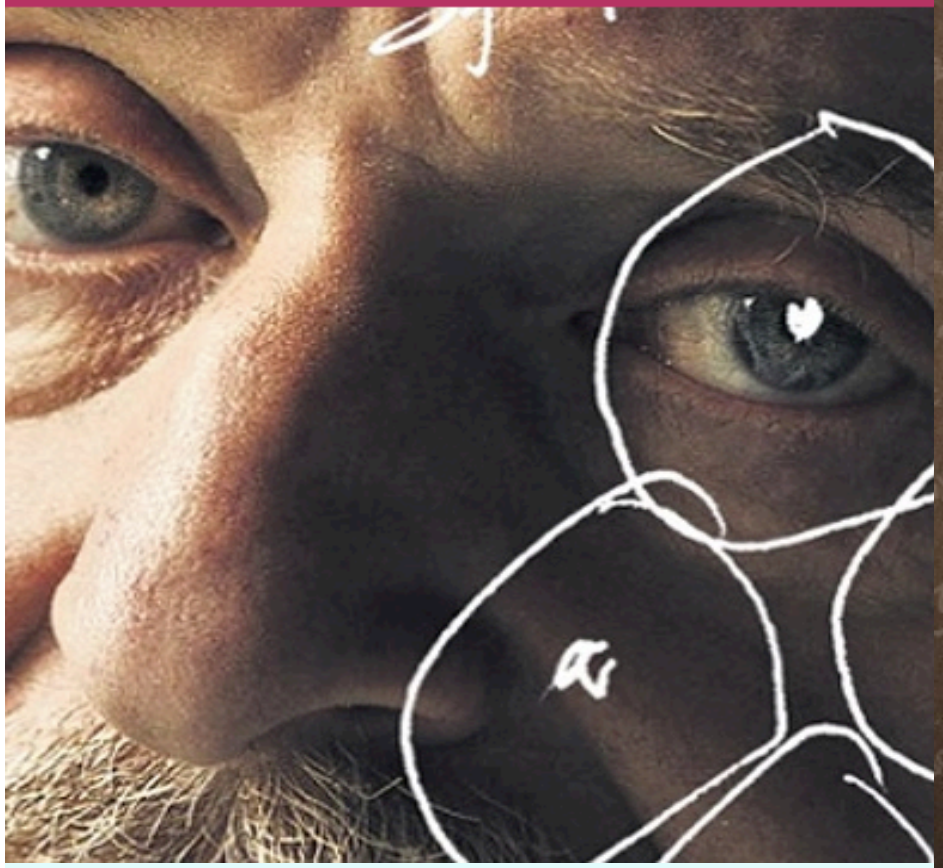
Finally, after six months or so, you find the light switch, you turn it on, and suddenly it's all illuminated. You can see exactly where you were. Then you move into the next room ….

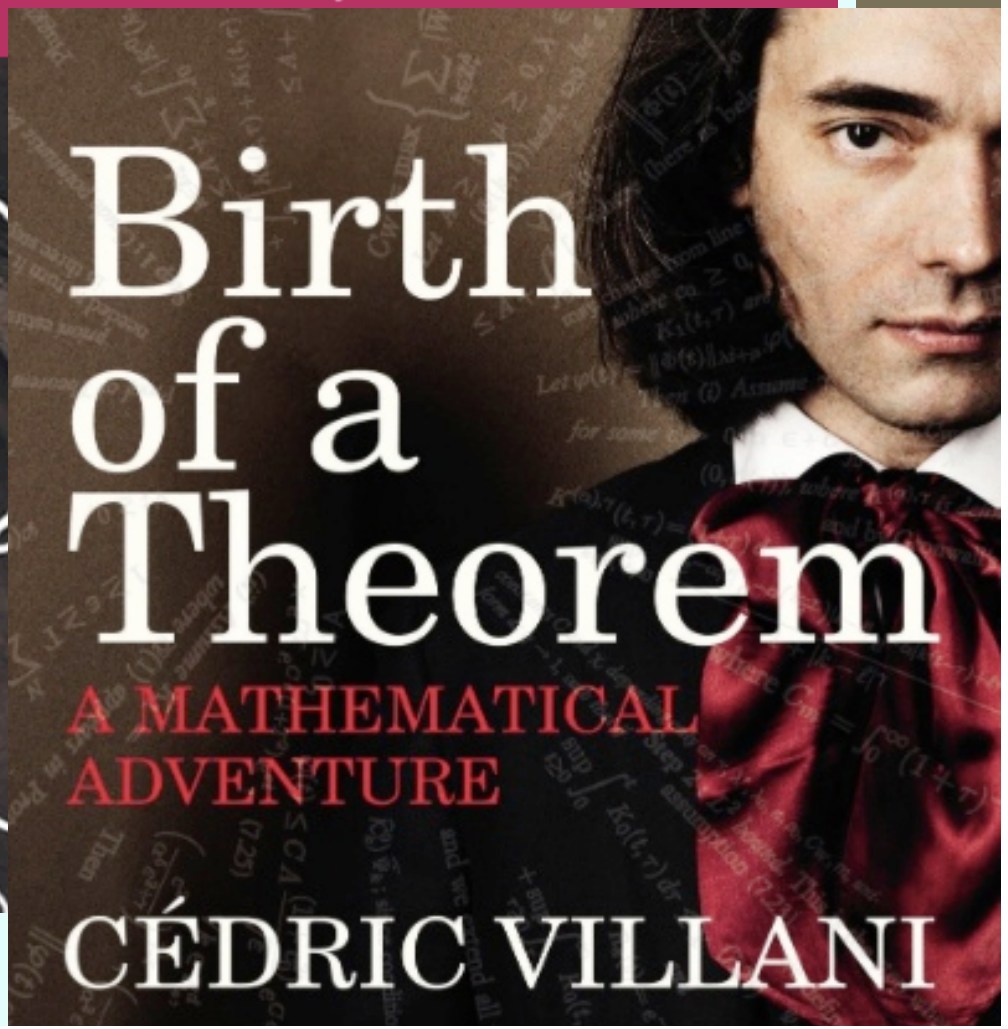
So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of – and couldn't exist without – the many months of stumbling around in the dark that precede them."

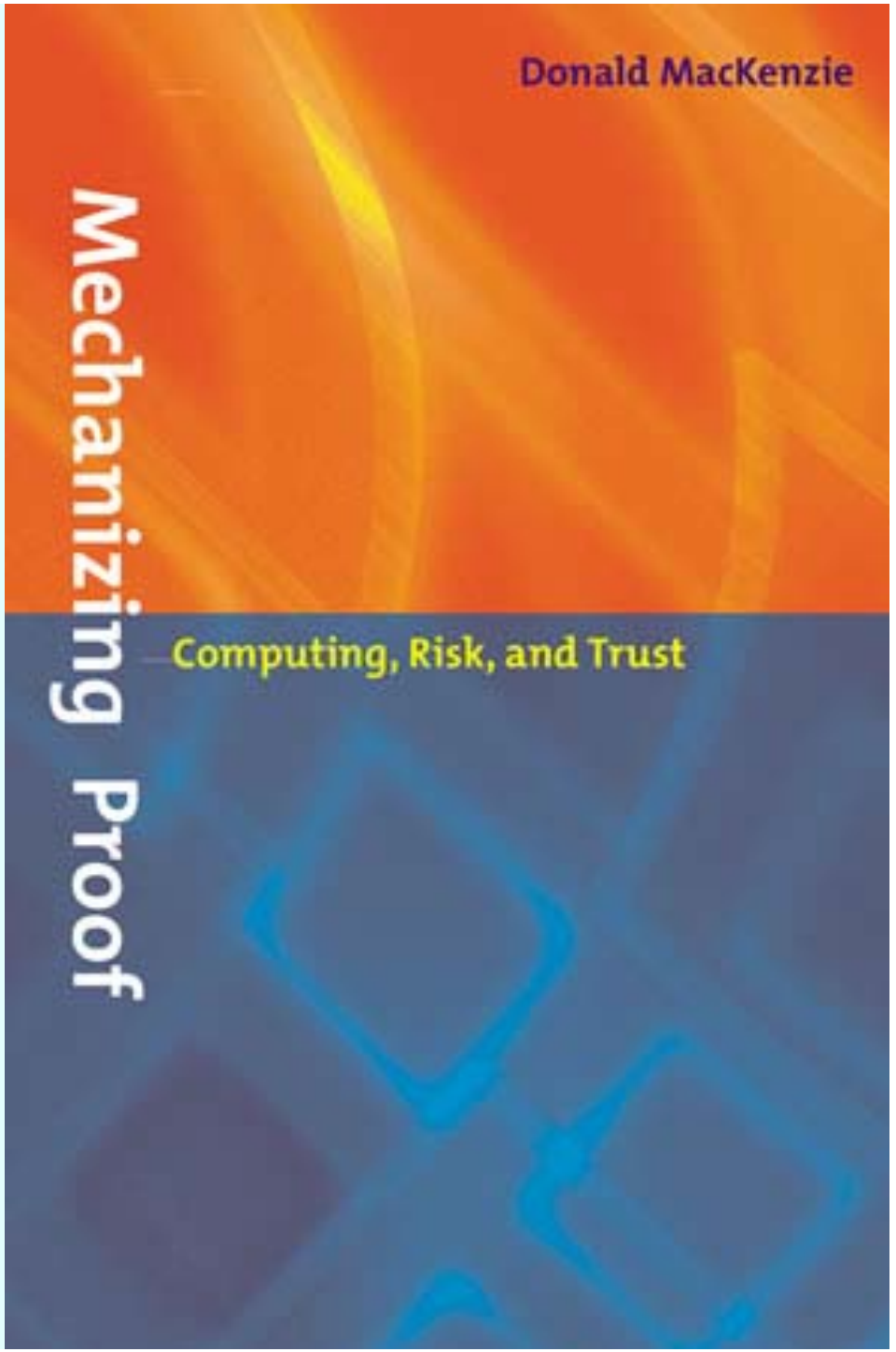
Andrew Wiles, 2000, on proving Fermat

John Horton Conway: the world's most charismatic mathematician

John Horton Conway is a cross between Archimedes, Mick Jagger and Salvador Dalí. For many years, he worried that his obsession with playing silly games was ruining his career – until he realised that it could lead to extraordinary discoveries






Donald MacKenzie
Mechanizing Proof
Computing, Risk, and Trust

… it is we who allow the machines' operations to count as correct deductions or deem them to be in error.

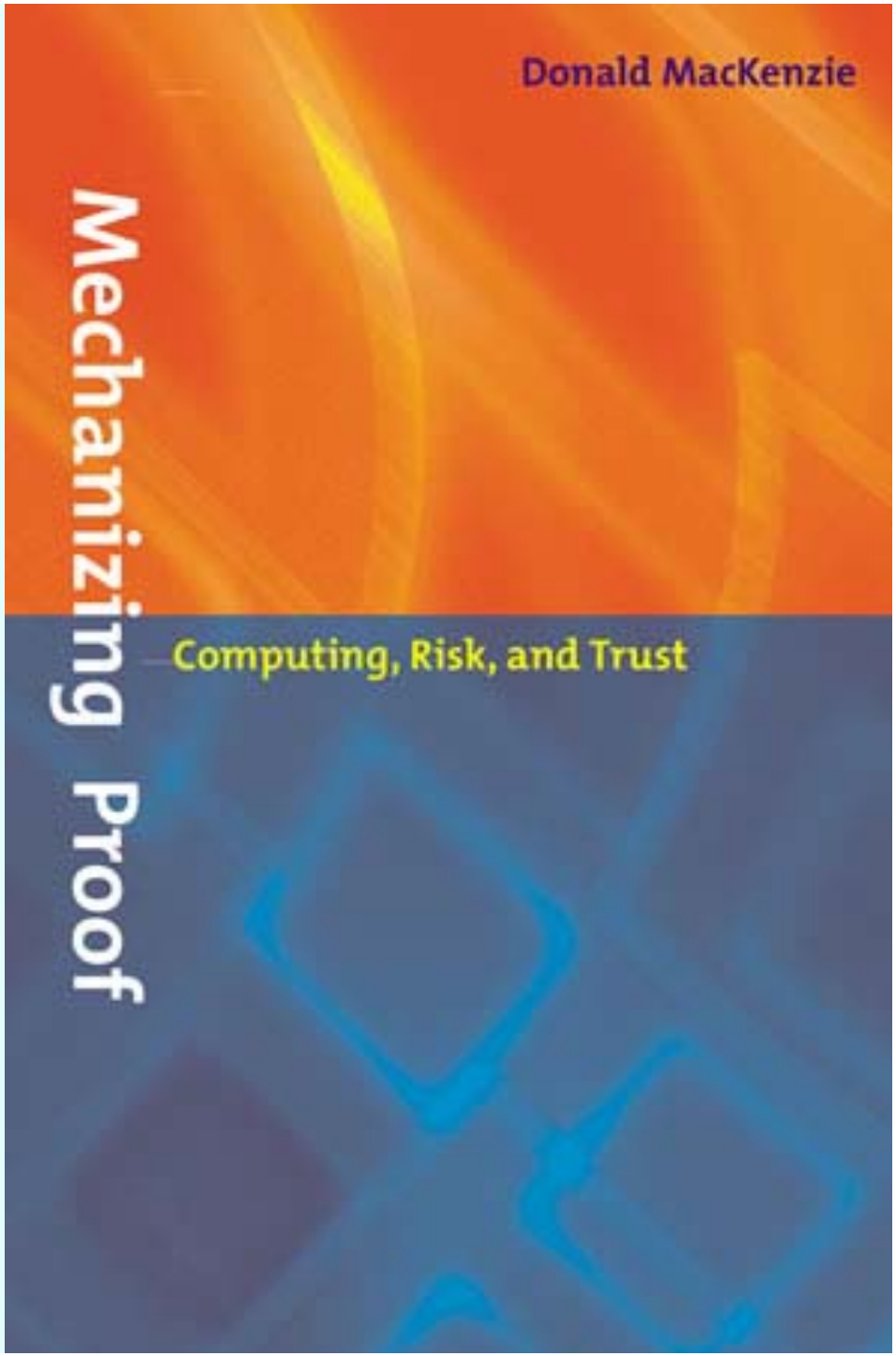
…

trust in the machine cannot entirely replace trust in the human collectivity.

Donald Mackenzie, Mechanizing proof, 2000

…too much informality risks uninventing proofs..

Alma Steingart 2012



Quillen K-theory

$\mathcal{M}$ exact cat (e.g. abelian)

called c.e.s. ← a family of ex seq's

$K_i\mathcal{M} = \pi_i(B\mathcal{M}^+)$ (Kan)

$= \pi_{i-1}(\Omega\mathcal{M})$

if $\mathcal{M} = f.g.$ pr. R-mods

$i \geq 0$

elem. gen's & rels

$K_0\mathcal{M}$ = Groth gp of $\mathcal{M}$

= gp's $[M]$ $M \in \mathcal{M}$

rels $[M] = [M'] + [M'']$

ker, coker

formalization

coq

trimaker

patched

HoTT

github

UniMath

3320 { VV – Foundations
AKS – RezkCompletion
G – Ktheory

no Higher Ind Types

Σ in prod's exist

sums

$\sum_i M_i \xrightarrow{\text{inj}} \prod_i M_i$

$(\hom(M,N)$ — a coset)

it's a ab gp

$\ker(M \to N) = \{ L \to M \mid f \circ g = 0 \}$

$K \to M$

$F \simeq \text{Yoneda}_K$

faster: consider rep'bility to mean that $El(F)$ has an initial object

where $F : \mathcal{M} \to \text{Sets}$

$El(F)$ obj: (L, x) $L \in \text{ob}\,\mathcal{M}$ $x \in F(L)$

(L', x') L $x' \in F(L')$

cat of ab gp's

gen's & rel'n's for ab gp's

"sum quot"

gen's $(i, x) \in I \times M$.

$(i, 0) = 0$

$(i, x+x') = (i, x) + (i, x')$

$p\ i=i' \in G$

$(i, x) = (i', \text{transp}_p\, x)$

interpretation I conn M — loc const. shf of ab gps

$\prod M_i$ = the gp of sections

$= (M_0)^G$

$\sum M = (M_0)_G$

a set is finite

a subset is an eq class

$P : \mathcal{U}$

P is a prop

Propositional Truncation

$X \to \|X\|$ (type) $x \mapsto |x|$ (prop) P (prop)

$|x| = |y|$

"merely X" $:\equiv \|X\|$

$\mathcal{M}$ add $\Rightarrow \mathcal{M}_{\text{merely}}$ has a zero obj

$\Rightarrow \forall M, N\ \exists 0 : \text{Hom}(M, N)$

$M \to N \to 0$

$P \equiv \sum_{y:Y} \prod_{x:X} fx = y$

Lemma $f : X \to Y$ Y set $\forall x, x' : X\ fx = fx'$

$\exists f' : \|X\| \to Y$ s.t. $f'|x| = fx$

1. $B\mathbb{Z}$ $\mathbb{Z} \simeq \|T\|$

$\Omega(T) \to$ (pf of $\simeq$)

The same idea gives the circle in HoTT without HIT.

G a group

Let BG

Thm $\Omega BG \simeq G$

Claim $B\mathbb{Z}$ is a circle — circle recursion

$B\mathbb{Z} \to$

a G-torsor is a set X w. an action $G \times X \to X$ s.t. $\forall x : X$ $G \xrightarrow{g \mapsto gx} X$ is an equiv & s.t. X is nonempty

the type of G-torsors. How a base pt appl'n of univalence

G w. left mult gives a torsor ✓ trivial torsor Triv

$\text{Aut}(\text{Triv}) = G$

$\downarrow$ UNIV

Identities — pairs

Mathematical practice: collaboration and polymath

Polymath collaborative projects

Massively collaborative mathematics

The 'Polymath Project' proved that many minds can work together to solve difficult mathematical prob
Timothy Gowers and **Michael Nielsen** reflect on the lessons learned for open-source science.

On 27 January 2009, one of us — Gowers — used his blog to announce an unusual experiment. The Polymath Project had a conventional scientific goal: to attack an unsolved problem in mathematics. But it also had the more ambitious goal of doing mathematical research in a new way. Inspired by open-source enterprises such as Linux and Wikipedia, it used blogs and a wiki to mediate a fully open collaboration. Anyone in the world could follow along and, if they wished, make a contribution. The blogs and wiki functioned as a collective short-term working memory, a conversational commons for the rapid-fire exchange and improvement of ideas.

The collaboration achieved far more than

that relied on heavy mathematical machinery. An elementary proof — one that starts from first principles instead of relying on advanced techniques — would require many new ideas. Second, DHJ implies another famous theorem, called Szemerédi's theorem, novel proofs of which have led to several breakthroughs over the past decade, so there is reason to expect that the same would happen with a new proof of the DHJ theorem.

The project began with Gowers posting a description of the problem, pointers to background materials and a preliminary list of rules for collaboration (see go nature

approximately 800 substantive com
containing 170,000 words. No one w
cifically invited to participate: anybod
graduate student to professional mat
tician, could provide input on any
Nielsen set up the wiki to distil notable i
from the blog discussions. The project r
commentary on at least 16 blogs, reac
front page of the Slashdot technolog
aggregator, and sp
a closely related
on Tao's blog. Thin
smoothly: neither I
'trolls' — persistent
of malicious or pu
fully distracting co

"Who would have guessed that the working record of a mathematical project would read like a thriller?"

Nature 461,15 October 2009

A NEW PROOF OF THE DENSITY HALES-JEWETT THEOREM

D. H. J. POLYMATH

ABSTRACT. The Hales–Jewett theorem asserts that for every r and every k there exists n such that every r-colouring of the n-dimensional grid $\{1,\dots,k\}^n$ contains a combinatorial line. This result is a generalization of van der Waerden's theorem, and it is one of the fundamental results of Ramsey theory. The theorem of van der Waerden has a famous density version, conjectured by Erdős and Turán in 1936, proved by Szemerédi in 1975, and given a different proof by Furstenberg in 1977. The Hales–Jewett theorem has a density version as well, proved by Furstenberg and Katznelson in 1991 by means of a significant extension of the ergodic techniques that had been pioneered by Furstenberg in his proof of Szemerédi's theorem. In this paper, we give the first elementary proof of the theorem of Furstenberg and Katznelson, and the first to provide a quantitative bound on how large n needs to be. In particular, we show that a subset of $\{1,2,3\}^n$ of density δ contains a combinatorial line if n is at least as big as a tower of 2s of height $O(1/\delta^2)$. Our proof is surprisingly simple: indeed, it gives arguably the simplest known proof of Szemerédi's theorem.

1. INTRODUCTION

1.1. **Statement of our main result.** The purpose of this paper is to give the first elementary proof of the density Hales–Jewett theorem. This theorem, first proved by Furstenberg and Katznelson [FK89, FK91], has the same relation to the Hales–Jewett theorem [HJ63] as Szemerédi's theorem [Sze75] has to van der Waerden's theorem [vdW27]. Before we go any further, let us state all four theorems. We shall use the notation $[k]$ to stand for the set $\{1,2,\dots,k\}$. If X is a set and r is a positive integer, then an *r-colouring* of X will mean a function $\kappa\colon X\to[r]$. A subset Y of X is called *monochromatic* if $\kappa(y)$ is the same for every $y\in Y$.

We begin with van der Waerden's theorem.

Blog plus social conventions

- Be polite and constructive
- Make your comments as easy to understand as possible
- It's OK for a mathematical thought to be tentative, incomplete, or even incorrect
- Excessively technical details should be placed on the wiki, or at another offsite location
- If you are planning to think about some aspect of the problem offline for an extended length of time, let the rest of us know
- An ideal polymath research comment should represent a "quantum of progress"

I do not believe that this is possible. Since the rotation is unstopped and always counterclockwise, the line will inevitably sweep through the unbounded space outside the convex hull of the points when a full rotation has taken place.

1 1 Rate This

Comment by Seungly Oh — July 19, 2011 @ 9:25 pm

> I believe Joel hopes to show that the area *not swept* is always bounded (or get a counterexample).
>
> 0 0 Rate This
>
> *Comment by Srivatsan Narayanan — July 19, 2011 @ 9:28 pm*
>
> > I think it is always bounded and lies inside the convex hull. I believe that is graphically obvious. But how does this help?
> >
> > 0 0 Rate This
> >
> > *Comment by Seungly Oh — July 19, 2011 @ 9:36 pm*
> >
> > > Oh yes, it does seem so. And yes, I am not sure it would help either.
> > >
> > > 0 0 Rate This
> > >
> > > *Comment by Srivatsan Narayanan — July 19, 2011 @ 9:40 pm*

7 February, 2009 at 8:56 am
Terence Tao

213. Upper and lower bounds

I added a table to the main post to reflect the progress so far on c_n, c'_n, c''_n for n up to 7. (I may expand the table up to n=10 or so at some later point.)

0 0 Rate This
Reply

7 February, 2009 at 9:21 am
Jason Dyer

(Meta) Quick fix: you put 1115 when it's 1155 for c_7

0 0 Rate This
Reply

7 February, 2009 at 9:28 am
Sune Kristian Jakobsen

214. Upper and lower bounds.

Shouldn't the upper bound for c_5 be 156? And I think the next upper bounds should be higher too.

0 0 Rate This
Reply

7 February, 2009 at 9:45 am
Terence Tao

Thanks for the corrections! I decided to go ahead and extend the table to n=10, though I am sure some of the bounds here could be optimised further.

0 0 Rate This
Reply

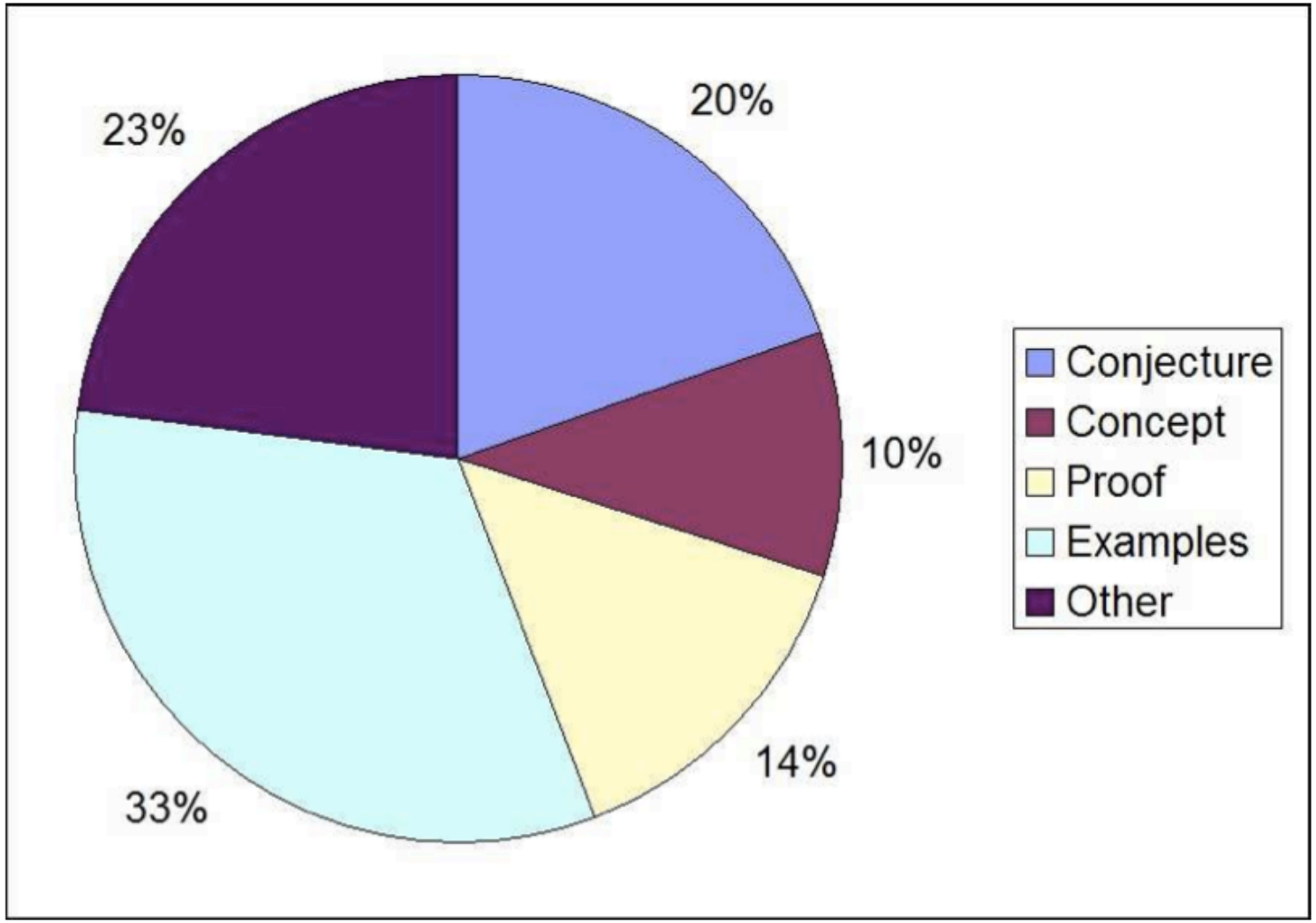
7 February, 2009 at 10:13 am
Sune Kristian Jakobsen

215. Algorithm / Idea for bounding the $|A|, |B|, |C|$ in 210

This is almost a copy of 340:

Analysing mini-polymath

Martin and Pease, 2012



Polymath 8

tinyurl.com/ktdjgkb

Improve the bound on the least gap between consecutive primes that is attained infinitely often, by developing the techniques of Zhang.

Twin primes conjecture:

∃ infinitely many prime pairs 3,5; 5,7; … 41,43; …

Zhang:

∃ infinitely many prime pairs p_1,q_1; p_2,q_2; …

with all $(q_i - p_i) < 70{,}000{,}000$

Tao et al via polymath: bound reduced to 246

Mathematical practice: experimental mathematics

Role of experimental mathematics

Gaining insight and intuition

Visualizing math principles

Discovering new relationships

Testing and especially falsifying conjectures

Exploring an idea to see if it merits more work

Suggesting approaches for proof

Computing replacing lengthy hand derivations

Confirming analytically derived results

(David Borwein and Jon Bailey)

Putting it all together: social machines

Model process – social machines

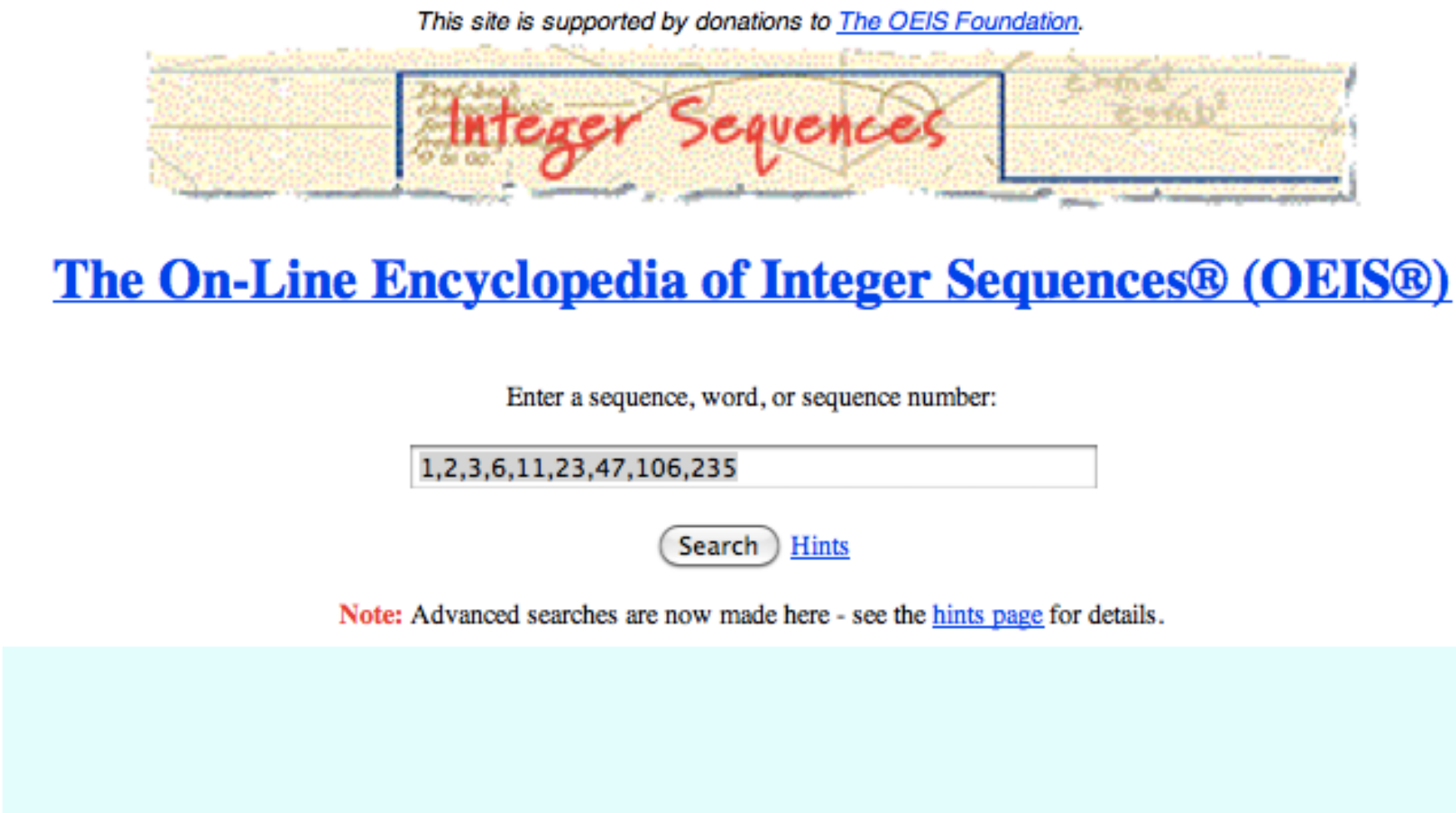
The Order of Social Machines

Real life is and must be full of all kinds of social constraint – the very processes from which society arises. Computers can help if we use them to create abstract social machines on the Web: processes in which the people do the creative work and the machine does the administration… The stage is set for an evolutionary growth of new social engines.

Berners-Lee, *Weaving the Web*, 1999

A social machine - OEIS

http://oeis.org/



Integer Sequences

1,2,3,6,11,23,47,106,235 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:1,2,3,6,11,23,47,106,235**

Displaying 1-1 of 1 result found. page 1

Sort: relevance | references | number | modified | created Format: long | short | data

A000055 Number of trees with n unlabeled nodes. +20
(Formerly M0791 N0299) 100

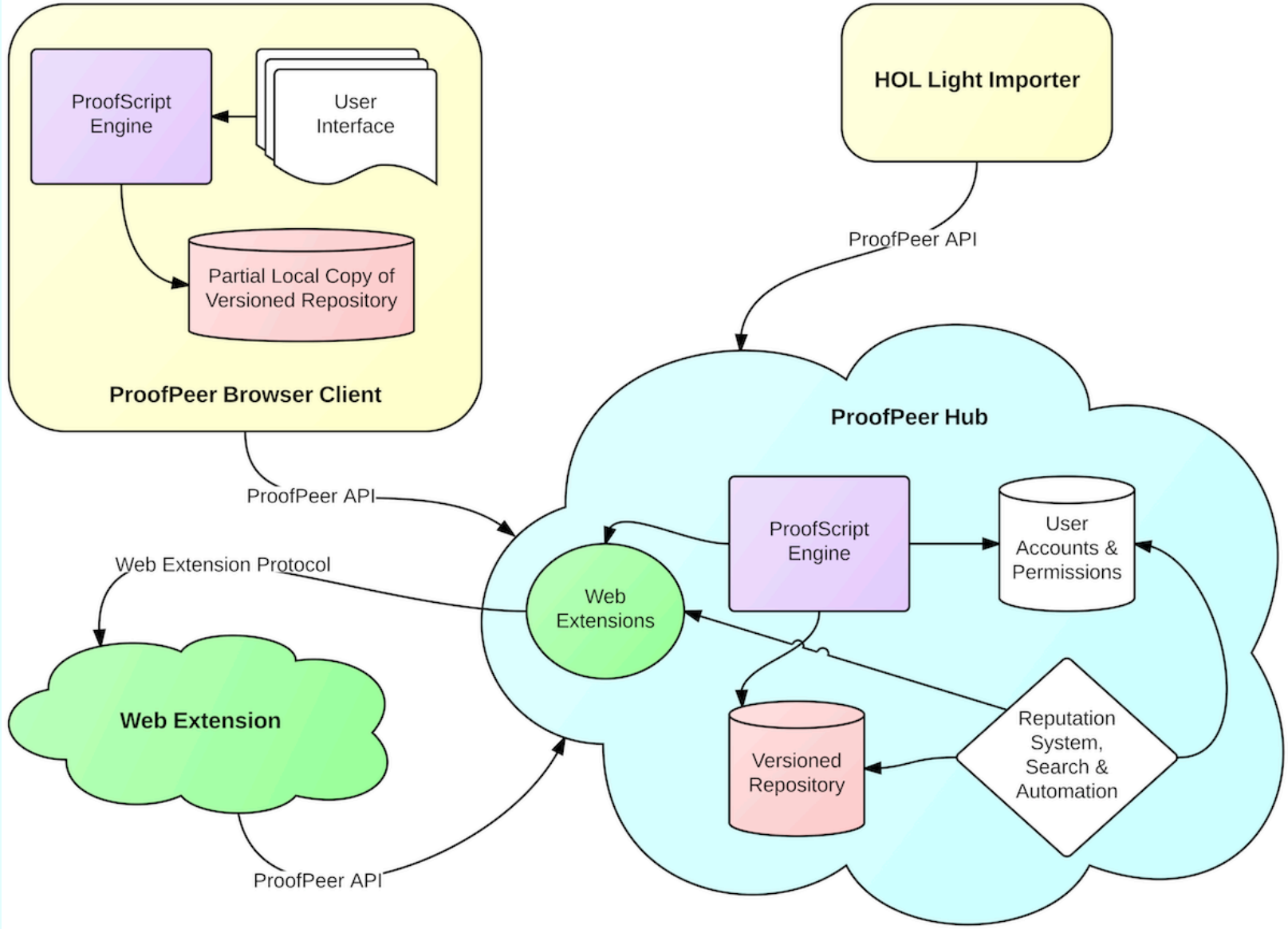
1, 1, 1, **1, 2, 3, 6, 11, 23, 47, 106, 235**, 551, 1301, 3159, 7741, 19320, 48629, 123867, 317955, 823065, 2144505, 5623756, 14828074, 39299897, 104636890, 279793450, 751065460, 2023443032, 5469566585, 14830871802, 40330829030, 109972410221 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,5

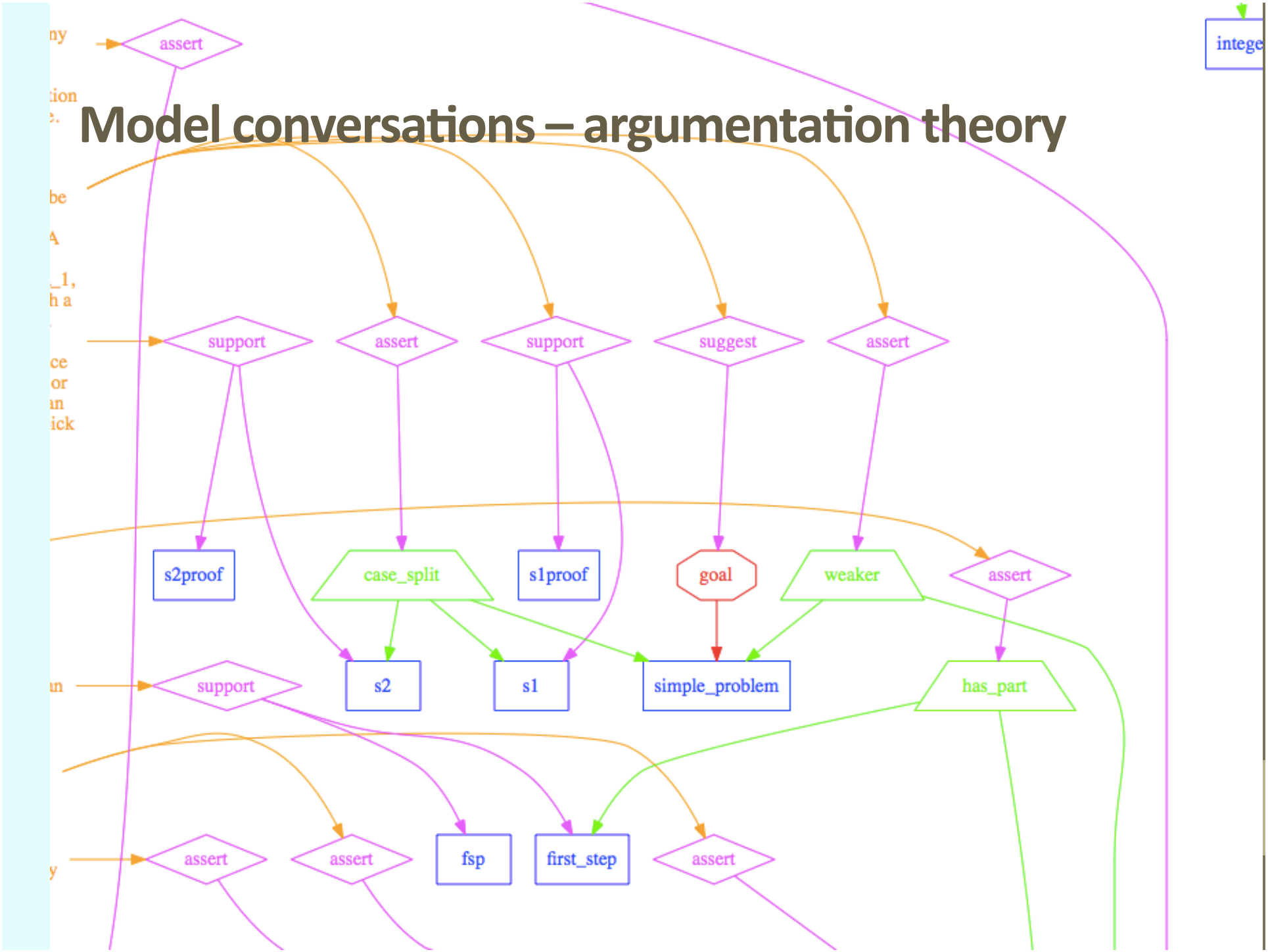
COMMENTS
Also, number of unlabeled 2-gonal 2-trees with n 2-gons.
Equals INVERTi transform of A157904: (1, 2, 4, 8, 17, 36, 78, 170,...). [From Gary W. Adamson, Mar 08 2009]
Equals left border of triangle A157905 [From Gary W. Adamson, Mar 08 2009]
Contribution from Robert Munafo, Jan 24 2010: (Start)
Also counts classifications of K items that require exactly N-1 binary partitions; see Munafo link at A005646, also A171871 and A171872.
The 11 trees for N = 7 are illustrated at the Munafo web link.
Link to A171871/A171872 conjectured by Robert Munafo, then proved by Andrew Weimholt and Franklin T. Adams-Watters on Dec 29 2009. (End)

REFERENCES
F. Bergeron, G. Labelle and P. Leroux, Combinatorial Species and Tree-Like Structures, Camb. 1998, p. 279.
N. L. Biggs et al., Graph Theory 1736-1936, Oxford, 1976, p. 49.
A. Cayley, On the analytical forms called trees, Amer. J. Math., 4 (1881), 266-268.
A. Cayley, On the analytical forms called trees, with application to the theory of chemical combinations, Reports British Assoc. Advance. Sci. 45 (1875), 257-305 = Math. Papers, Vol. 9, 427-460 (see p. 459).
S. R. Finch, Mathematical Constants, Cambridge, 2003, pp. 295-316.
Andrew Gainer-Dewar, Gamma-Species and the Enumeration of k-Trees,

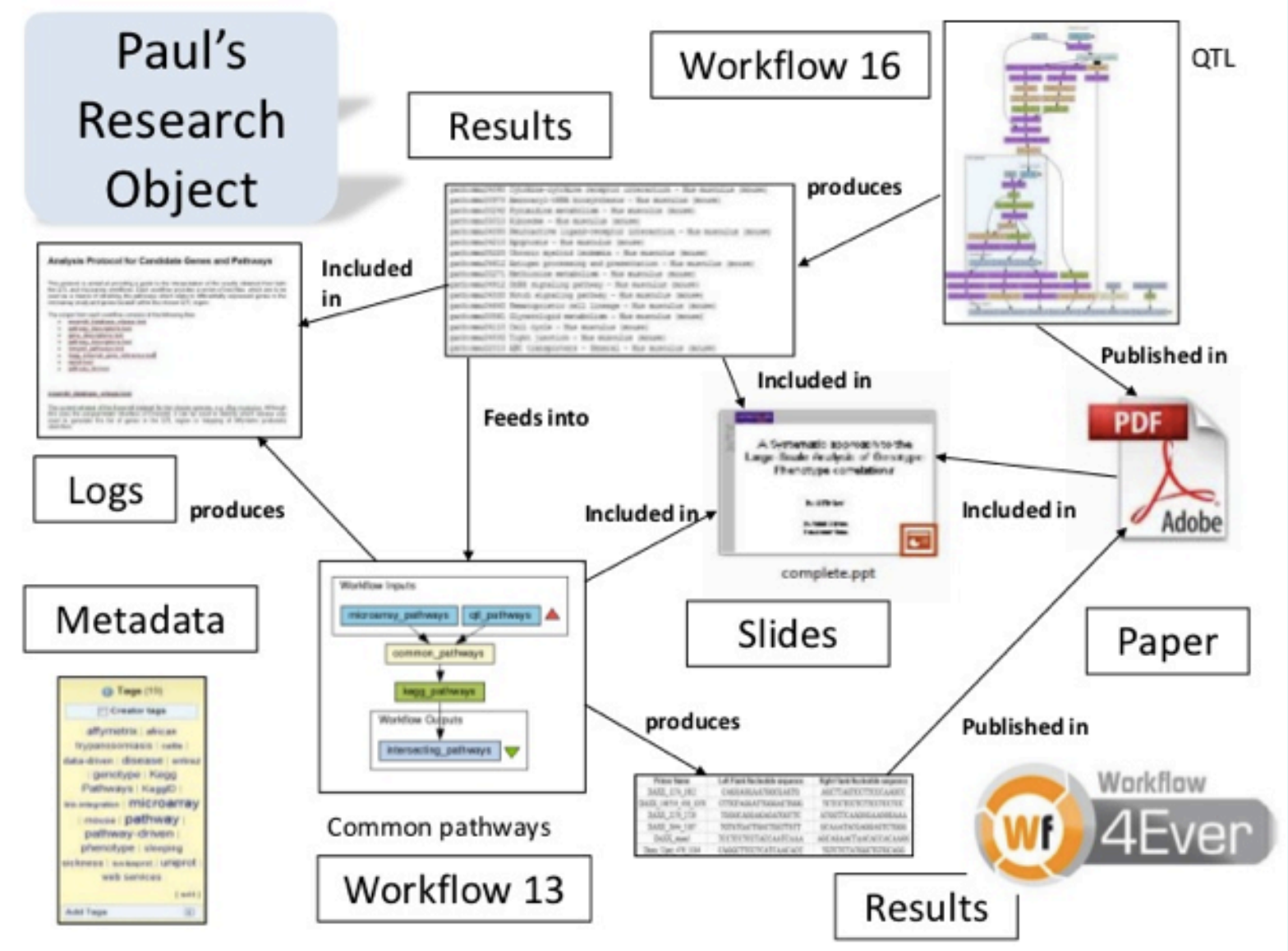
Proof peer - collaborative theorem proving Fleuriot and Obua



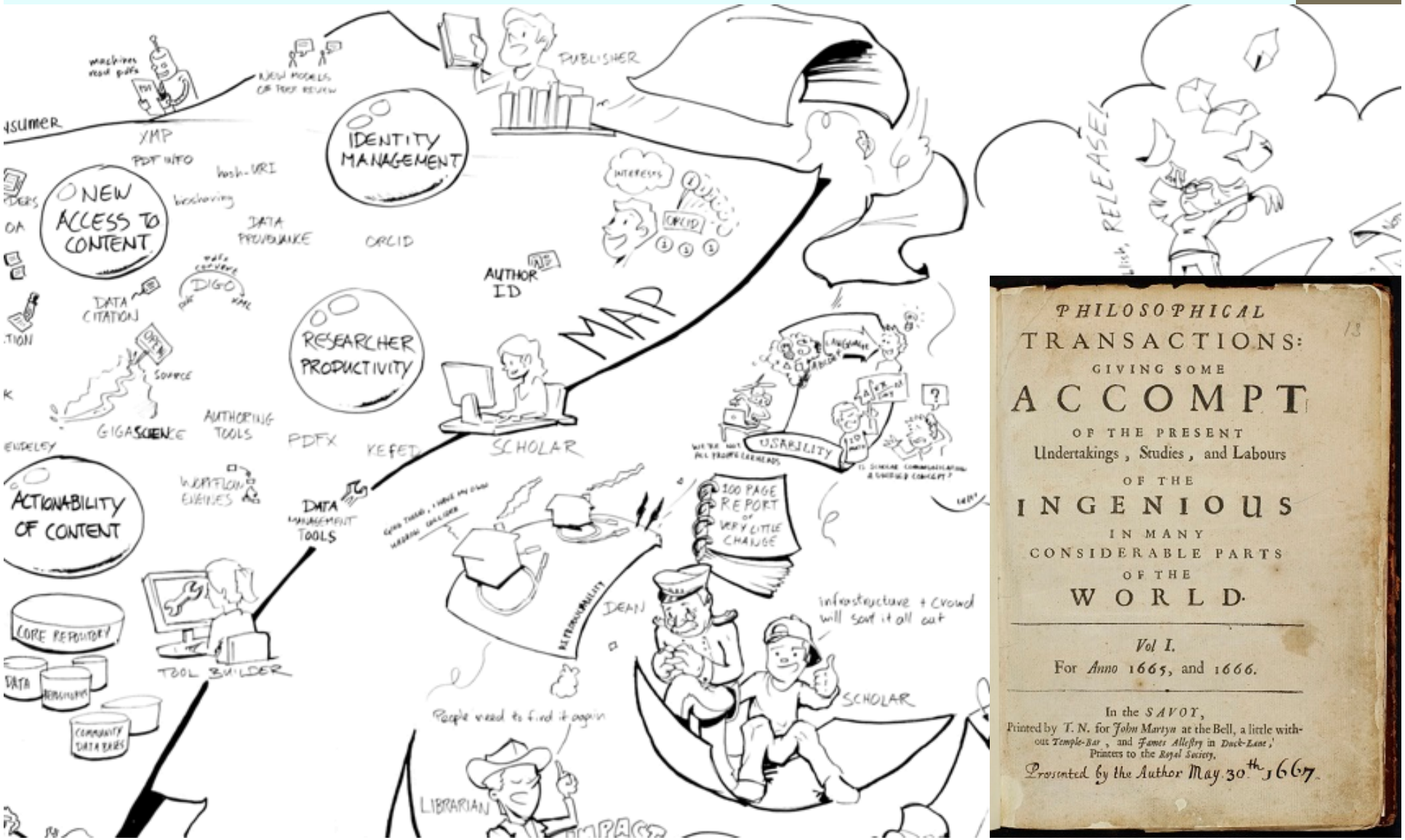
Model conversations – argumentation theory



Scientific workflow



Mathematical publication 2050??

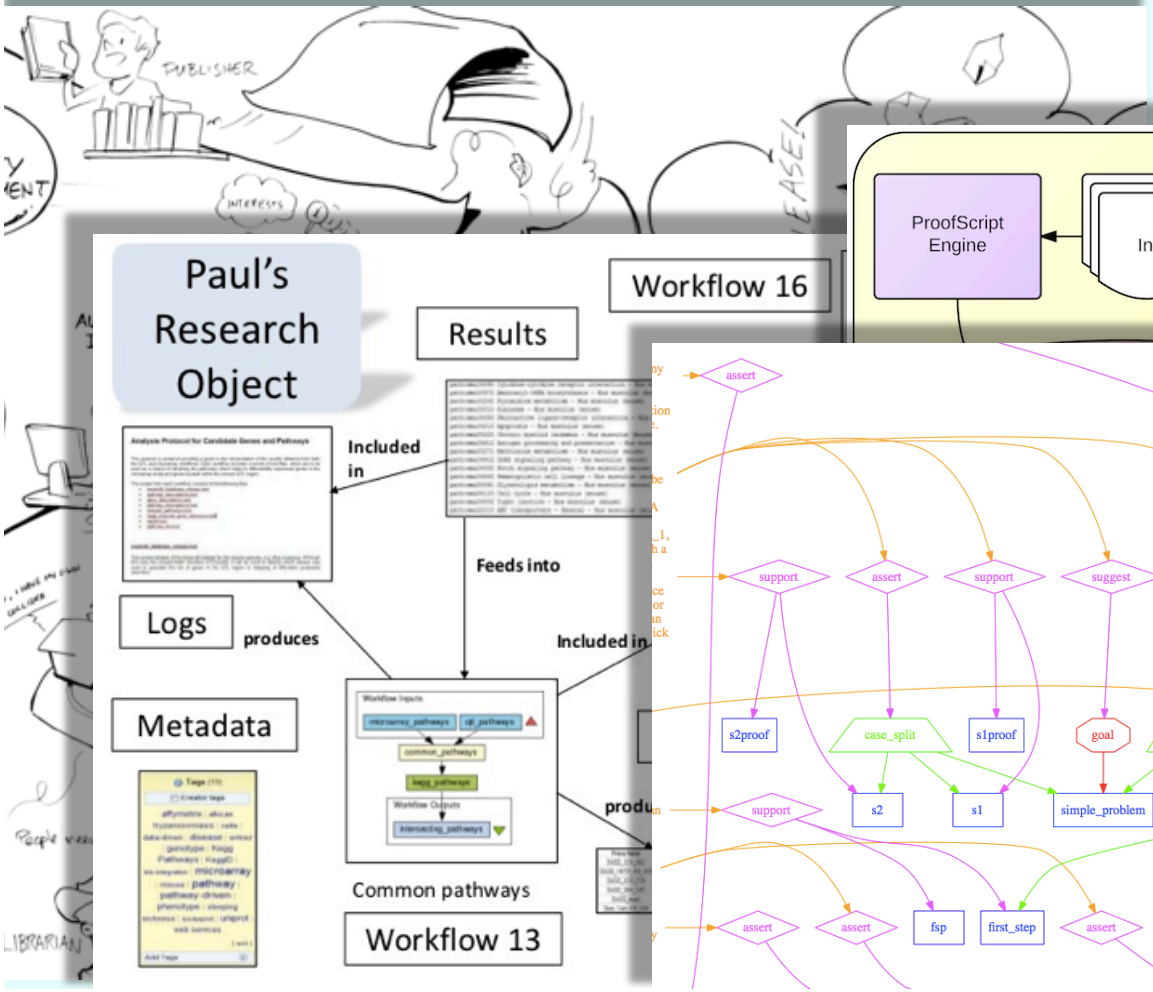


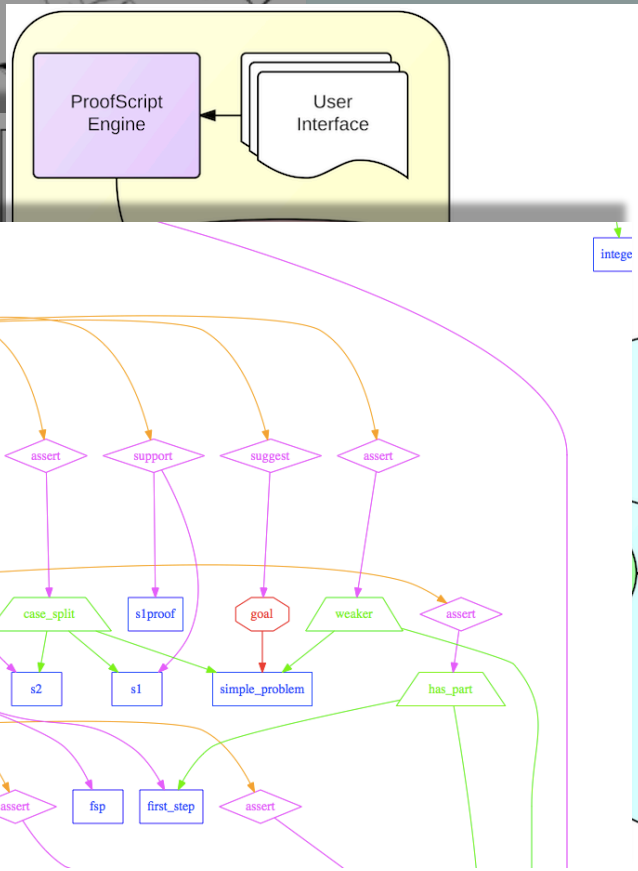
PHILOSOPHICAL
TRANSACTIONS:
GIVING SOME
ACCOMPT
OF THE PRESENT
Undertakings, Studies, and Labours
OF THE
INGENIOUS
IN MANY
CONSIDERABLE PARTS
OF THE
WORLD.

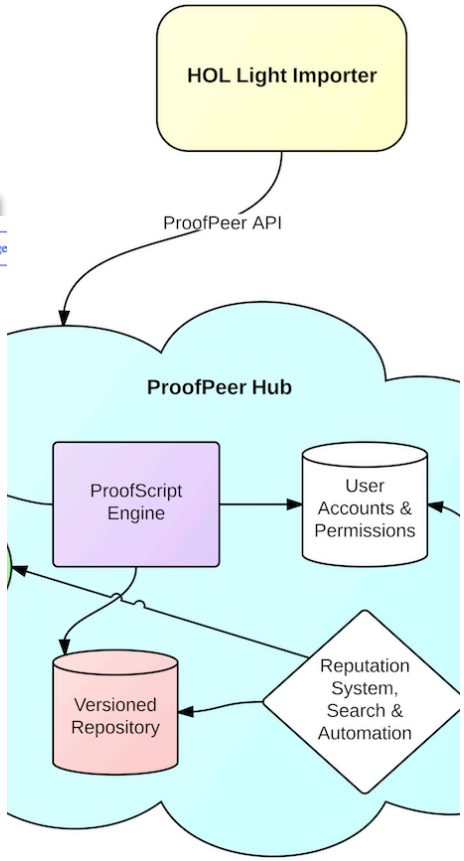
Vol I.
For *Anno* 1665, and 1666.

In the *SAVOY*,
Printed by *T. N.* for *John Martyn* at the Bell, a little without *Temple-Bar*, and *James Allestry* in *Duck-Lane*,
Printers to the *Royal Society*.
Presented by the Author May 30th 1667.

Challenge for CADE 2050:
Create the social machine of mathematics







Looking back…..

Looking back.....

CADE 0

Symposium on Automatic Demonstration,

Rocquencourt, France, 1968

Springer Lecture Notes in Mathematics 125

On the long-range prospects of automatic theorem-proving

Hao Wang

There is a false contrast between the algorithmic and the heuristic approaches. Every program has to embody some algorithm and for serious advances, partial strategies or heuristic methods are indispensible. Hence, no serious program could avoid either component. Perhaps the contrast is more between anthropomorphic and logicist, as typified by the general problem solver on the one hand and elaborate refinements of the Herbrand theorem on the other. This polarization appears to me to be undersirable and to represent what I would call the reductionist symptom.

Typically the reductionist is struck by the power or beauty of certain modes to proceed and wish to build up everything on them. The two extremes seem to share, in practice if not in theory, this reductionist preoccupation. In my opinion, there should be more reflective examination of the data, viz. the existing mathematical proofs and methods of proof. It is true that what is natural for man need not be natural or convenient for machine. Hence, it will not be fruitful to attempt to imitate man slavishly. Nevertheless, the existing body of mathematics contains a great wealth of material and constitutes the major source of our understanding of mathematical reasoning.

Congratulations CADE, on your 25th (or 26th) anniversary, and best wishes for the next 25 (or 24 or 26) conferences!